Combinatorics 1

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$${}^nC_k = \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$n! = \prod_{\substack{i \in \mathbb{N} \\ i \le n}} (i) = n(n-1)(n-2)\ldots 1$$

$${}^nP_k = \frac{n!}{(n-k)!}$$

Q: n objects. How many ways we can choose exactly 1 object

Ans: n

Q: How many " " " " 2 objects

Ans:-

to choose 1^{st} object we have n options

to choose the 2^{nd} object we have $n-1$ options

Total number of options in the process $= \sum_{\text{for all } n \text{ objects}} (n-1) = n(n-1) = {}^nP_2$

$\{1,2\}$
$\{2,1\}$ repeated 2 times

Total number of ways $= \frac{n(n-1)}{2} = {}^nC_2 = \binom{n}{2}$

Q:- How many ways to have exactly 3 objects

Ans:- 1^{st} object $\rightarrow n$
2^{nd} object $\rightarrow n-1$
3^{rd} object $\rightarrow n-2$

$$\longrightarrow \sum_n \left(\sum_{n-1} (n-2) \right) = \sum_n (n-1)(n-2) = n(n-1)(n-2)$$

$\{1, 2, 3\} \Rightarrow 3!$

No. of way to choose $= \frac{n(n-1)(n-2)}{6} = \frac{n!}{(n-3)!\,3!} = {}^nC_3$

$$\frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = \frac{n!}{(n-3)!}$$

Without order choosing k objects from n we have ${}^nC_k$ ways

With order " " " " ${}^nP_k$ "

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5 objects are distinct

_ _ _ _ _ _ _
1 2 3 4 5 6 7

1^{st} object options $= 7$
2^{nd} " " $= 6$
⋮
5^{th} " " $= 3$

$$7 \times 6 \times 5 \times 4 \times 3 = \frac{7!}{2!} = {}^7P_5$$

with repetitions

2 | | 1 | 4 | 3 | | 5
1 | | 2 | 5 | 4 | | 3

_ _ _ _ _ _ _

1 2 3 4 5 6 7

5 objects not distinct

1st object option = 7.

2nd " " = 6

⋮

5th " " = 3

Total options $= 7\times 6\times 5\times 4\times 3 = \frac{7!}{2!}$

For each case 5! arrangements are similar

Total combinations $= \frac{7!}{2!\,5!} = {}^7C_5$

4th object fixed

4 object distinct

_ _ 4 _ _ _

pos: 1 2 3 4 5 6

1st object = 5

2nd object = 4

3rd object = 3

Total options $= 5\times 4\times 3$
$= {}^5P_3$

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4 objects not distinct

_ _ 4 _ _ _

1 2 3 4 5 6

Total options $= 5\times 4\times 3 = {}^5P_3$

Total arrangements $= \frac{5\times 4\times 3}{3!} = {}^5C_3$